Calculated cosmic ray pion and proton fluxes at sea level

1975 J. Phys. A: Math. Gen. 8 1530

(http://iopscience.iop.org/0305-4470/8/9/023)

J. Phys. A: Math. Gen., Vol. 8, No. 9, 1975. Printed in Great Britain. © 1975

Calculated cosmic ray pion and proton fluxes at sea level

Keran O'Brien

Health and Safety Laboratory, United States Energy Research and Development Administration, New York 10014, USA

Received 25 March 1975, in final form 9 May 1975

Abstract. A critical analysis is made of recent calculations of the sea level cosmic ray proton and pion spectra expected at ground level and comparison is made with two sets of experimental data measured at Durham. Particular attention is directed towards the theoretical approach advanced by the present author in a previous paper and some misconceptions by Hook and Turver are pointed out. The work of Jabs is also considered. It is concluded that all three sets of calculations represent the data equally well.

1. Introduction

In presenting the results of some Monte Carlo calculations of sea level pion and proton spectra, Hook and Turver (1974) also review the calculations of Jabs (1968, 1972) and of O'Brien (1971a). The review contains a number of errors, and as neither the calculations of Jabs (1968, 1972) nor those of O'Brien are compared with the data directly, it is difficult to form a correct understanding of the relative significance of the three sets of results.

For these reasons, it was felt worthwhile to describe the theory used by O'Brien and to compare all three sets of calculations with both the measurements performed at Durham during the period 1960–2 (Brooke and Wolfendale 1964, Brooke *et al* 1964) and with the new measurements performed in January–February 1971 (Diggory *et al* 1974). It will be apparent from the quality of the data (the measurement difficulty, particularly in the case of the sea level pion spectrum, is severe) that there is little to choose among the three sets of theoretical results.

2. Theory

2.1. The stationary Boltzmann equation

The behaviour of cosmic rays in the earth's atmosphere, averaged over time, is governed by the stationary form of the Boltzmann equation:

$$B_q\varphi_q(r, E, \mathbf{\Omega}) = S_{qj} \tag{1.1}$$

$$B_q = \mathbf{\Omega} . \nabla + \sigma_q + \frac{C_q}{p_q r} - \frac{\partial}{\partial E}k_q \tag{1.2}$$

$$S_{qj} = \int_{4\pi} d\mathbf{\Omega}' \int_E^{\infty} dE_B \sigma_{qj} F_{qj}(E_B \to E, \mathbf{\Omega}' . \mathbf{\Omega})\varphi_j(r, E_B, \mathbf{\Omega}') \tag{1.3}$$

$$C_q = m_q H / c\tau_q$$

where

- r is the depth in the atmosphere in g cm^{-2};
- E is the particle kinetic energy in MeV;
- $\boldsymbol{\Omega}$ is the unit vector in the direction of particle travel;
- φ_q is the particle flux of type q per MeV per second per steradian at a depth r having a direction $\boldsymbol{\Omega}$;
- σ_q is the total cross section for the absorption of a particle of type q in cm^2 g^{-1};
- k_q is the stopping power of a charged particle of type q in air in MeV cm^2 g^{-1};
- σ_{qj} is the cross section for the production of particles of type q from collisions with or decay by particles of type j in cm^2 g^{-1};
- F_{qj} is the number of q-type particles per MeV per second per steradian at E and $\boldsymbol{\Omega}$ resulting from collisions with a j-type particle at E_B and $\boldsymbol{\Omega}'$;
- m_q is the rest mass of a q-type particle in MeV c^{-2};
- p_q is the momentum of a q-type particle in MeV c^{-1};
- τ_q is the mean life of a q-type particle in its rest frame in seconds;
- H is the scale height of the earth's atmosphere (taken as $6{\cdot}7 \times 10^5$ cm); and
- c is the velocity of light (3×10^{10} cm s^{-1}).

It is understood that k_q is zero for neutrons, σ_q is zero for muons and so forth.

O'Brien assumed, for the sake of extracting a solution of the Green-function type, that

$$F_{qj} = (1-l)K_q \frac{E_B^l}{E^{l+1}} U(E_B - \eta_q) \frac{\delta(1 - \boldsymbol{\Omega}' . \boldsymbol{\Omega})}{2\pi} \tag{2}$$

where

- l is an arbitrary constant to be determined by fitting to experimental yield data;
- K_q is the partial inelasticity for the production of a q-type particle from a collision with a primary particle of energy E_B;
- η_q is a constant, equal to 500 MeV. This will compensate to some degree for the omission of particle stopping in the case of protons and pions;
- U is the Heaviside function; and
- δ is Dirac's improper function.

2.2. *Comments on the solution and associated approximations*

The values of K_q were derived from Hagedorn and Ranft (1968), and the constant l was determined, by fitting the energy–angle charged particle production spectra (equation (2)) to the shower particle multiplicity data reviewed by Meyer *et al* (1963), to be 0·216. The constant l was not chosen to optimize the fit to the sea level cosmic ray data. It is noted in passing that the pion and proton production spectra F_{qj} were also compared with the then current models by O'Brien. However, no optimization, no choice of parameter based on improving the agreement of calculated cosmic ray flux, spectrum or collision density with experiment was carried out.

Kaon transport, not kaon production, was neglected in O'Brien's (1971a) treatment. In that paper, reference is made to an earlier publication (O'Brien 1970) where the effect of kaon transport was studied, and the conclusion reached that its effect on ground level cosmic rays was negligible. The effect of kaons on the ground level muon spectrum was considered in some detail a little later (O'Brien 1971b), and the same conclusion reached.

The stationary Boltzmann equation (equation (1)) contains no explicit provision for the introduction of fluctuations in the elasticity coefficient. However, the integral in the right-hand side of the Boltzmann equation (the 'scattering-down integral') incorporates secondary production spectra. The secondary nucleon spectra used in these calculations are essentially equivalent to the elasticity spectrum.

O'Brien's calculations are only one-dimensional in the sense that the straight-ahead approximation is used (this is a consequence of the Dirac function in equation (2)). However, these calculations have a considerably broader scope than the calculation of vertical sea level cosmic ray spectra, and have been applied to the calculation of scalar neutron flux and neutron density, and of ionization, dose and particle spectra at large zenith angles. Typically, agreement with measured data was within about 20%.

Lastly, the two sources of the primary cosmic ray spectrum used by O'Brien were clearly identified: Freier and Waddington (1965) below 10 GeV per nucleon, and Peters (1958) above 10 GeV per nucleon.

3. Comparison with experiment

In figure 1, the measurements of the proton spectrum by Diggory *et al* (1974) and by Brooke and Wolfendale (1964), and the measurements of the negative pion spectrum

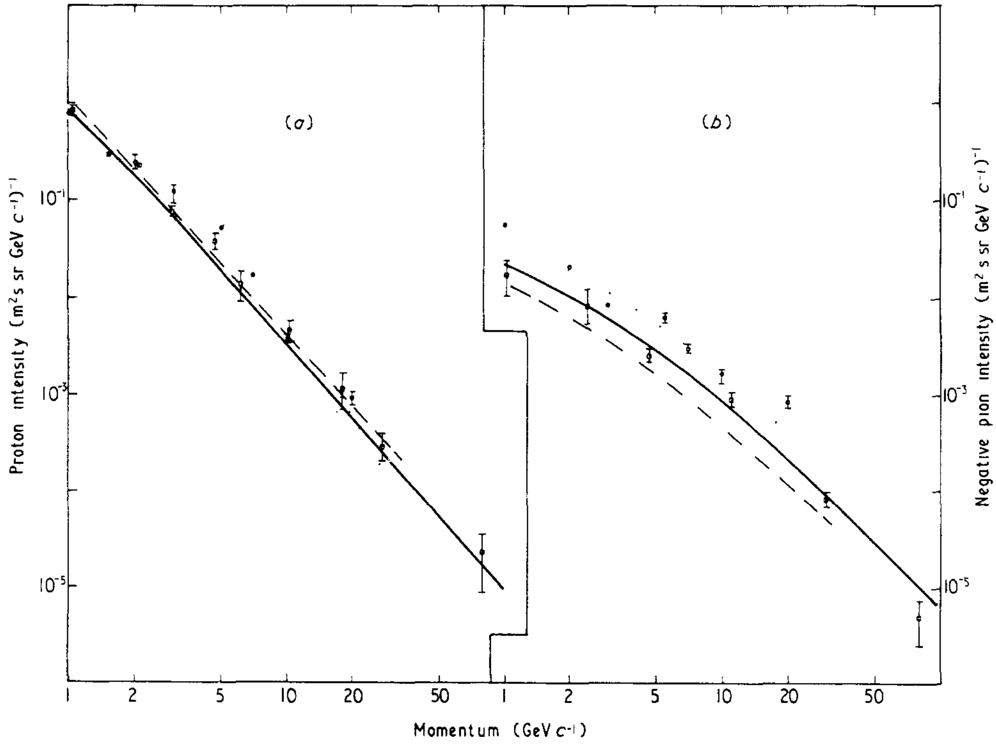

Figure 1. Calculated and measured proton and pion spectra at sea level. The calculations were carried out by: Jabs (1968, 1972) (broken curve); O'Brien (1971) (full curve); and Hook and Turver (1974) (dotted curve). The measurements were made by: ⊙ Diggory *et al* (1974); ⊡ Brooke and Wolfendale (1964) (*a*); and ⊡ Brooke *et al* (1964) (*b*).

by Diggory *et al* (1974) and by Brooke *et al* (1964) are compared with the calculations of O'Brien, of Jabs and of Hook and Turver. The quality of the data of these difficult experiments does not allow a choice among the three sets of calculations.

Essentially the same is the case with respect to the pion spectrum, with the added complication that the later results of Diggory *et al* (1974) appear to be systematically higher than the results of Brooke *et al* (1964) by about 50%. A comparison of figure 1 of this paper with figure 7 of Hook and Turver (1974) seems to indicate that Jabs' results have been misplotted.

Finally, figure 2 shows the pion to proton ratio calculated by O'Brien and as measured by Diggory *et al* (1974), Brooke *et al* (1964) and Subramanian (1962). In this case the agreement with the data is satisfactory.

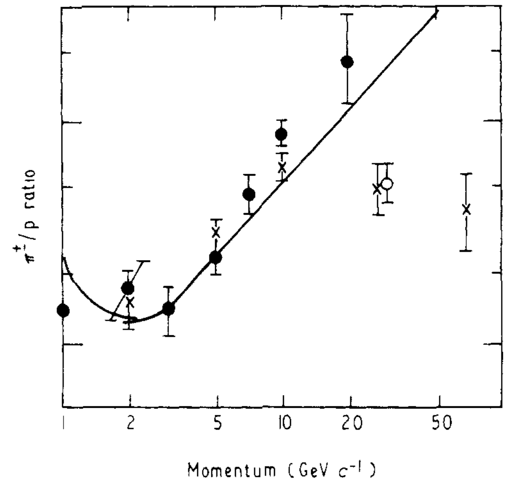

Figure 2. Calculated and measured values of the pion to proton ratio in the sea level cosmic ray spectrum. The calculations were carried out by O'Brien (1971) (full curve). The measurements were made by. × Brooke *et al* (1964); ○ Subramanian (1962); and ● Diggory *et al* (1974).

No particular allowance was made for solar modulation in the comparison of these calculations with experiment, although it should be observed that the earlier Durham measurements were made during a period of strong solar activity, which would compensate to some degree for the effect of the 20% error in the use of the original Rossi value to determine the aperture of the spectrometer.

O'Brien's calculations were carried out for a zenith angle of 0°. For this study, the spectra were also calculated at 10°, corresponding to the estimated effective zenith angle of the data of Diggory *et al*. The 10° calculations are about 15% lower than the 0° calculations.

4. Conclusions

This paper has directly compared the calculations of Hook and Turver, Jabs and O'Brien with both the new and the old sets of sea level hadron spectra measured at

Durham. While differing in detail among themselves, the three sets of calculations appear to describe the experimental data about equally well.

The status of the experimental pion spectrum is somewhat obscured by the fact that the pion spectrum measured by Diggory *et al* is about 50% higher than the earlier spectrum measured by Brooke *et al.* It has not been possible to assign a reason for this discrepancy. It should be noted that both sets of Durham measurements of the proton and pion spectra are of 'unaccompanied' particles. The term 'unaccompanied' in this sense is not well defined, and its meaning will differ from experiment to experiment. This could possibly account for part of the discrepancy between the two sets of pion measurements. Comparison of such measured intensities with calculation is not straightforward due to the difficulty in allowing for the 'unaccompanied' requirement.

References

Brooke G, Meyer M A and Wolfendale A W 1964 *Proc. Phys. Soc.* **83** 871–7
Brooke G and Wolfendale A W 1964 *Proc. Phys. Soc.* **83** 843–52
Diggory I S, Hook J R, Jenkins I A and Turver K E 1974 *J. Phys.* A: *Math., Nucl. Gen.* **7** 741–64
Freier P S and Waddington C J 1965 *Proc. 9th Int. Conf. on Cosmic Rays, London* vol 1 (London: The Institute of Physics and the Physical Society) pp 176–9
Hagedorn R and Ranft J 1968 *Nuovo Cim. Suppl.* **6** 169–310
Hook J R and Turver K E 1974 *J. Phys.* A: *Math., Nucl. Gen.* **7** 765–78
Jabs A 1968 *Z. Phys.* **212** 222–34
—— 1972 *Nuovo Cim.* A **12** 569–610
Meyer H, Teucher M W and Lohrmann E 1963 *Nuovo Cim.* **28** 1399–411
O'Brien K 1970 *J. Geophys. Res.* **75** 4357–9.
—— 1971a *Nuovo Cim.* A **3** 521–47
—— 1971b *Phys. Rev.* D **4** 635–8
Peters B 1958 *Handbook of Physics* eds E U Condon and H Odishaw (New York: McGraw-Hill) pp 9/207–45
Subramanian A 1962 *PhD Thesis* University of Madras